Laws of Exponents

There are only four major laws of exponents that determine all that we do with exponents and exponential functions.

Assume a and b are positive real numbers, while x and y are any real numbers (possibly negative or irrational)

1. $a^{x+y} = a^x a^y$
2. $a^{x-y} = \frac{a^x}{a^y}$
3. $(a^x)^y = a^{xy}$
4. $(ab)^x = a^x b^x$

Notice the second law implies $a^{-x} = \frac{1}{a^x}$, so we can think of a negative in an exponent as "sending that piece to the denominator".

These rules can also be combined to form more complicated statements. Here are some examples:

$$\left(\frac{5^8 * 5^3}{5^{16}}\right)^3 = (5^{11-16})^3 = (5^{-5})^3 = 5^{-5*3} = 5^{-15}$$

We will need to be able to cancel complicated exponential expressions quickly and accurately (especially for infinite series).

$$\frac{3^{3n+7}}{5^{4-2n}}\frac{5^{2(3-n)}}{3^{3(2+n)}} = \frac{3^{3n+7}}{3^{3(2+n)}}\frac{5^{2(3-n)}}{5^{4-2n}} = \frac{3^{3n+7}}{3^{3n+6}}\frac{5^{6-2n}}{5^{4-2n}} =$$

$$3^{3n+7-(3n+6)}5^{6-2n-(4-2n)} = 3^1 5^2 = 75$$

We also need to be able to reorganize exponential functions so that the role of the variable is clear.

$$4^{3n-2}7^{5-2n} = 4^{-2}7^5 4^{3n}7^{-2n} = 4^{-2}7^5(4^3)^n(7^{-2})^n =$$

$$\frac{7^5}{4^2}(4^3 7^{-2})^n = \frac{7^5}{16}\left(\frac{4^3}{7^2}\right)^n = \frac{7^5}{16}\left(\frac{64}{49}\right)^n$$