Factoring Rules

Here are a few basic factoring examples which occur often enough to justify memorizing them. Again, recognition will be the key to using these rules in Calculus.

1. $a^2 - b^2 = (a-b)(a+b)$
2. $(a+b)^2 = a^2 + 2ab + b^2$
3. $(a-b)^2 = a^2 - 2ab + b^2$
4. $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
5. $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
6. $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

As you might expect, the last three are used less often than the first three, since they involve third powers. In fact, number 1 is probably the most important, since we often run into the difference of two perfect squares, especially in geometric problems.

The middle four examples can also be written as part of Pascal's triangle, but we'll discuss that (a bit) more in class.

Completing the Square

Given $x^2 + bx$ or $x^2 + bx + c$, it is sometimes useful to rewrite it in the form $(x+a)^2 + d$ using rule 2 (or 3 if b is negative), for some constants a and d related to b (and c). This is called completing the square, and it's where the quadratic formula comes from.

Let $a = \frac{b}{2}$ so $b = 2a$. Then $x^2 + bx = x^2 + 2ax$ which looks like two of the three pieces of rule 2. The last piece of rule 2 would be adding a^2, but to keep things fair we must also subtract a^2 so we don't change anything. This gives us

$$x^2 + bx = x^2 + 2ax = (x^2 + 2ax + a^2) - a^2 = (x+a)^2 - a^2$$

so $a = \frac{b}{2}$ and $d = -a^2 = -\left(\frac{b}{2}\right)^2$.

If we started with $x^2 + bx + c$, then the c just comes along for the ride and we get

$$x^2 + bx + c = x^2 + 2ax + c = (x^2 + 2ax + a^2) - a^2 + c = (x+a)^2 + (c - a^2)$$

so $a = \frac{b}{2}$ didn't change, but now $d = c - a^2 = c - \left(\frac{b}{2}\right)^2$.

Examples:

$$x^2 + 6x + 7 = (x^2 + 2(3)x + 9) + (7 - 9) = (x+3)^2 - 2$$

$$x^2 - 5x + 3 = x^2 - 2\frac{5}{2}x + \frac{25}{4} + \left(3 - \frac{25}{4}\right) = \left(x - \frac{5}{2}\right)^2 + \frac{12 - 25}{4} = \left(x - \frac{5}{2}\right)^2 - \frac{13}{4}$$