

# SPECTRAL ANALYSIS ON SINGULAR AND NONCOMPACT MANIFOLDS

AMS Meeting in Tucson AZ, April 21-22, 2007

DAN BURGHELEA

**TITLE: Elliptic theory on orbifolds (after Bogdan Bucicovshi).**

**ABSTRACT:** Given the present interest in the geometric analysis on orbifolds I will present the key contributions of B. Bucicovschi to spectral geometry of elliptic operators in vector orbibundles over orbifolds. They concern mostly the Seeley theory of complex powers. B. Buchicovshi died in 2004 and his work was not published. A preliminary version of his work can be recovered from arXiv, DG 9912228v1. *Received:* February 24, 2007

THALIA JEFFRES

**TITLE: Vertical blow-ups for prescribed mean curvature equations on nonconvex domains.**

**ABSTRACT:** Following Professor Lancasters resolution of the long-outstanding Concus-Finn Conjecture for capillary surfaces over convex domains with a corner, in joint work we are pursuing some extensions of these methods to domains that are nonconvex. *Received:* February 21, 2007

CHRIS JUDGE

**TITLE: Spectral simplicity of the Laplacian on generic polygons.**

**ABSTRACT:** We consider the Laplacian acting on either Dirichlet or Neumann functions on a simply connected polygon with  $n > 4$  vertices. We show that, for almost every polygon, the eigenvalues of this operator are simple. We will also discuss generalizations to non-simply connected domains, other manifolds, and higher dimensions. *Received:* February 04, 2007

KLAUS KIRSTEN

**TITLE: Functional determinants for separable partial differential operators.**

**ABSTRACT:** Functional determinants of partial differential operators are of great importance in many applications in mathematical physics. For example they arise in semiclassical approximations in quantum mechanics and quantum field theory. As a preparation for the higher dimensional considerations we will first consider second order self-adjoint elliptic systems of differential operators on the one-dimensional interval. For general boundary conditions, a contour integral method for the calculation of the related functional determinants is provided. Afterwards partial differential operators are analyzed for cases where eigenfunctions can be obtained using separation of variables. Cartesian and polar coordinates serve as examples.

In particular we perform a dimensional reduction by which results for partial differential operators are given explicitly in terms of a series of determinants of an associated ordinary differential operator. *Received:* January 26, 2007

MATTHIAS LESCH

**TITLE: Regularized traces and  $K$ -theory invariants of parametric pseudodifferential operators.** **ABSTRACT:** I will outline our recent construction of invariants of relative  $K$ -theory classes of multi-parameter dependent pseudodifferential operators, which recover and generalize Melrose's divisor flow. These are 'higher' divisor flows, that are obtained by pairing relative  $K$ -theory classes with relative cyclic cocycles manufactured out of regularized traces. They take integral values and can be interpreted as 'suspended' versions of the spectral flow. This talk is based on joint work with Henri Moscovici and Markus Pflaum. *Received:* February 28, 2007

VLADIMIR MAZ'YA

**TITLE: Derivatives of biharmonic functions near nonsmooth boundary.** **ABSTRACT:** The following three results concerning solutions of the Dirichlet problem for a biharmonic equation will be discussed. 1. Boundedness of the gradient of a solution for an arbitrary three-dimensional domain. 2. Boundedness of the Hessian of a solution on a convex domain (all dimensions). 3. Necessary and sufficient conditions for continuity of the gradient at a point of the boundary of an arbitrary three-dimensional domain. This is a joint work with S. Mayboroda (OSU). *Received:* February 22, 2007

RAFE MAZZEO

**TITLE: Elliptic operators on quasi-asymptotically conic spaces.** **ABSTRACT:** The quasi-asymptotically Euclidean (QALE) spaces described by Joyce are of interest partly because they carry complete Ricci at metrics. The class of quasi-asymptotically conic (QAC) spaces is a natural generalization, and it is of interest to understand various aspects of elliptic theory on these, including Hodge, index and scattering theory. This is a report on the rst steps in this program. Joint work with Anda Degeratu. *Received:* February 26, 2007

PATRICK McDONALD

**TITLE: The eta invariant for quantum graphs.** **ABSTRACT:** Let  $G$  be a compact quantum graph and  $E$  an Hermitian vector bundle over  $G$ . Let  $C^\infty(G, E)$  be the functions  $\phi : G \rightarrow E$  which are smooth on the interior of each edge and which have a smooth extension to each closed edge. Given a complex structure on  $E$ , the length associated to each edge gives rise to a Dirac operator,  $D : C^\infty(G, E) \rightarrow C^\infty(G, E)$ . We parameterize self-adjoint extensions of  $D$  using a collection of unitary matrices determined by  $E$  and the given complex structure. For each self-adjoint extension of the Dirac operator we analyze the eta function and compute the eta invariant using the associated unitary matrix. *Received:* February 27, 2007

GERARDO MENDOZA

TITLE: **The  $b$ -Dolbeault complex.**

ABSTRACT: Let  $M$  be the closure of a smooth bounded domain in a complex manifold. There is an involutive subbundle  $V$  of  $\mathbb{C}^bTM$  whose sections are sections of  $T^{0,1}M$  thus giving a complex of  $b$ -operators on sections of the exterior powers of  $V^*$  closely related to the Dolbeault complex. While this is not a  $b$ -elliptic complex, it can be analyzed using techniques developed for  $b$ -elliptic operators. This gives a perspective of the classical Dolbeault complex from the point of view of the  $b$ -philosophy. *Received:* February 25, 2007

FREDERIC ROCHON

TITLE: **The residue determinant from a topological point of view.**

ABSTRACT: After reviewing various notions of determinants on spaces of pseudodifferential operators, we will give a topological description of the residue determinant recently introduced by Simon Scott. We will then indicate how this can be used to get a global existence result for this determinant. *Received:* February 22, 2007

JULIE ROWLETT

TITLE: **Spectral geometry and asymptotically conic convergence.**

ABSTRACT: We define asymptotically conic (AC) convergence in which a family  $\{g_\epsilon\}$  of smooth Riemannian metrics on a fixed compact manifold  $M$  degenerate to a singular metric  $g_0$  on a compact manifold with boundary  $M_0$ , where  $g_0$  has an isolated conic singularity at the boundary. This convergence is related to the analytic surgery metric degeneration of Mazzeo-Melrose and is the model problem for ongoing work of Degeratu-Mazzeo on QALE/QAC spaces. We discuss the general setup for AC convergence and present two spectral convergence results.

1. Convergence of the spectrum of geometric Laplacians for  $g_\epsilon$  to the spectrum of the Friedrichs extension of geometric Laplacian for  $g_0$ .
2. Asymptotic expansion in  $\epsilon$  of the corresponding heat kernels as  $\epsilon \rightarrow 0$ , with uniform convergence in  $t$ .

We summarize the techniques of the proofs which include rescaling arguments, parametrix construction on manifolds with corners, maximum principle, and a new resolution blowup and parameter ( $\epsilon$ ) dependent heat operator calculus developed in this work. *Received:* February 12, 2007

JOHN TOTH

TITLE: **Nodal lines of convex ergodic billiards.**

ABSTRACT: I will present recent results (joint with S. Zelditch) on asymptotic upper bounds for the number of zeros of boundary traces of Neumann eigenfunctions on convex, ergodic billiards. *Received:* February 21, 2007