MICROLOCAL ANALYSIS AND SPECTRAL THEORY ON SINGULAR SPACES

Penn State University, October 24-26, 2009

Pierre Albin

Title: **The signature package on Witt spaces.**

Abstract: I will report on joint work with Eric Leichtnam, Rafe Mazzeo, and Paolo Piazza on understanding the signature operator on a stratified manifold with C^*-algebra coefficients. This operator comes up naturally in the study of higher signatures of stratified manifolds.

Assuming the space is Witt we inductively construct a parametrix and prove the operator is Fredholm, extending results of Cheeger to the C^*-algebra context. I will also discuss work in progress identifying the index class of this operator with the 'symmetric signature' of the Witt space.

David Borthwick

Title: **Sharp upper bounds for resonance counting in perturbations of hyperbolic space.**

Abstract: For a class of "black box" perturbations P of the Laplacian on H^{n+1}, we derive an explicit constant B_P such that the resonance counting function satisfies $N_P(r) \leq B_P r^{n+1} + O(r^n \log r)$. This constant is sharp in the sense that for a single spherical obstacle in H^{n+1}, we have $N_p(r) \sim B_P r^{n+1}$.

Emily Dryden

Title: **Upper bounds for eigenvalues of submanifolds.**

Abstract: The problem of finding upper bounds for eigenvalues of the Laplace operator has a rich history, beginning with Hersch's sharp upper bound on λ_1 for the sphere. In contrast, Colbois and Dodziuk showed that for manifolds of dimension three and higher, the eigenvalues can be unbounded unless additional geometric constraints are imposed. We discuss upper bounds on eigenvalues in the setting of compact submanifolds of Euclidean space. Our bounds depend on the dimension and volume of the submanifold and the order of the eigenvalue, plus a geometric constraint which measures the "volume concentration" of the submanifold.

This is joint work with Bruno Colbois and Ahmad El Soufi.

Daniel Grieser

Title: **Pseudodifferential calculus for multiply fibred cusps.**

Abstract: We present a pseudodifferential calculus generalizing the 'fibred cusp calculus' introduced by Mazzeo and Melrose and generalized by Vaillant to the case of multiply fibred boundaries and corresponding cusp differential operators. In the case of two fibrations these are locally of the form $P(x, y, z, w; x^3\partial_x, x^2\partial_y, x\partial_z, \partial_w)$. Such operators arise for example as Laplace operator on locally symmetric spaces of Q-rank one. We give conditions for Fredholmness and prove boundedness results on suitable Sobolev spaces. This is joint work with E. Hunsicker.

Christopher Kottke

Title: **An Anghel-Callias index theorem and monopole charges.**

Abstract: I will discuss a class of index problems generalizing the one first investigated by C. Callias. His result concerned the index of a spin Dirac operator coupled to a skew-adjoint vector potential $D \otimes 1 + i \otimes \Phi$ on odd-dimensional Euclidean space, which turns out to depend only on the Chern character (in this case the topological winding number) of the potential Φ over the sphere at infinity. Subsequent generalization to more general odd-dimensional noncompact manifolds was obtained by N. Anghel.

My first result is a K-theory formulation and proof of an Anghel-Callias-type index theorem, permitting generalization to arbitrary self-adjoint elliptic pseudodifferential operators in the scattering calculus of Melrose, coupled to skew-adjoint potentials with appropriate invertibility and commutativity conditions at infinity. The index is entirely determined by symbolic and topological data over the manifold at infinity, and a families version of the theorem is also obtained.

Next I will discuss some work in progress concerning the index of such operators when the potential Φ is not invertible at infinity, but merely has constant rank. A perturbation argument reduces this to the full rank case, provided we can solve a certain related model problem.

Finally I will mention applications to the above to the moduli space of SU(2) monopoles over a class of manifolds, where the index of such an operator gives the dimension of the moduli.

Jun Masamune

Title: **Homogenization and its applications to the convergence of Wiener measures.**

Abstract: Homogenization is a powerful method to study the spectral structure of a singular space. The standard procedure is to find a sequence of non-singular spaces and to show an appropriate convergence of the energies of those spaces to that of the singular space. In particular, we are interested in the application of the homogenization in probability theory.

In this paper, we study a domain with singular measure, and show the Mosco convergence and the tightness of the associated Wiener measures. As a consequence, we obtain the weak convergence of the Wiener measures.

Svitlana Mayboroda

Title: **Properties of the biharmonic functions: Hadamard's conjecture, regularity of the Green function and Wiener criterion.**

Abstract: In 1908 Hadamard conjectured that the biharmonic Green function must be positive. Later on, several counterexamples to Hadamard's conjecture have been found and a variety of upper estimates were obtained in sufficiently smooth domains. However, the behavior of the Green function in general domains was not well-understood until recently.

In a joint work with V. Maz'ya we derive sharp pointwise estimates for the biharmonic and, more generally, polyharmonic Green function in arbitrary domains. Furthermore, we introduce the higher order capacity and establish an analogue of the Wiener criterion describing the precise correlation between the geometry of the domain and the regularity of the solutions to the polyharmonic equation.

Patrick McDonald

Title: **Hitting times and spectra for metric graphs.**

Abstract: Metric graphs, the simplest examples of singular spaces, are one dimensional piecewise linear spaces with singularities at the vertices. Metric graphs carry Laplace operators. The associated spectral theory of metric graphs has undergone extensive development, as has the theory of diffusions on metric graphs. Our results concern the relationship between the spectral theory of a compact metric graph and natural invariants associated to corresponding diffusions on the graph. These (nonspectral) invariants are constructed via hitting times for diffusions on the metric graph and are closely related to the heat content of the metric graph. For a certain class of metric graphs, we prove that our invariants determine the associated spectrum.

Gerardo Mendoza

Title: **The trace of the resolvent of an elliptic cone operator.**

Abstract: I will describe some aspects of the proof of a theorem obtained in collaboration with J. Gil and T. Krainer asserting the existence of an asymptotic expansion of the trace of the resolvent, and describing its structure, of a suitable closed extension of a general elliptic cone operator A in the presence of a ray of minimal growth for the principal symbols $({}^{c}\boldsymbol{\sigma}(A), A_{\wedge,\mathcal{D}_\wedge})$ of the operator. The primary focus will be on the differential-geometric analysis that leads to the structure of the resolvent for $A_{\wedge,\mathcal{D}_\wedge}$ along the ray.

Shahla Molahajloo

Title: **Pseudo-differential operators on the unit circle.**

Abstract: We give the basic properties of pseudo-differential operators with symbols in $S^m_{1,0}(\mathbb{S}^1 \times \mathbb{Z})$, $-\infty < m < \infty$, on $L^p(\mathbb{S}^1)$, $1 < p < \infty$. Ellipticity is then defined and the equivalence of Fredholmness on $L^p(\mathbb{S}^1)$ and ellipticity is established.

Frédéric Rochon

Title: **Ricci flow and the determinant of the Laplacian on non-compact surfaces.**

Abstract: After introducing the notion of determinant of the Laplacian on a non-compact surface with ends asymptotically isometric to a cusp or a funnel, we will show that in a given conformal class (with 'renormalized area' fixed), this determinant is maximal for the metric of constant scalar curvature, generalizing a well-known result of Osgood, Phillips and Sarnak in the compact case. This will be achieved by combining a corresponding Polyakov formula with some long time existence result for the Ricci flow for such metrics. This is a joint work with P. Albin and C.L. Aldana.

Julie Rowlett

Title: **Conformal deformations to constant negative scalar curvature for singular spaces – joint work with T. Jeffres.**

Abstract: We consider "cone-edge" singular spaces which are generalizations of singular orbifolds that allow both warping and any compact manifold to be the "link" of the singular set. For a cone-edge space with a Riemannian metric with negative scalar curvature, we determine necessary and sufficient conditions for the existence of a conformal deformation to a cone-edge space with constant negative scalar curvature. These conditions have a nice geometric interpretation. I will discuss examples of cone-edge spaces and metrics and their relation to other types of singular spaces, outline the key ingredients in our proofs, and give recipes for constructing two different types of constant negative scalar curvature cone-edge spaces.

Bert-Wolfgang Schulze

Title: **Norm growth estimates of parameter-dependent edge operators.**

Abstract: Operators on corner manifolds, degenerate in stretched variables (motivated by metrics when a corner configuration is embedded in a smooth ambient manifold) have a natural principal symbolic hierarchy, coming from the strata of the underlying space. Symbols of that kind are well- known in many special cases, e.g., (apart from the standard interior symbols on the main stratum) boundary symbols in boundary value problems, or conormal symbols in the conical case when the base is smooth. Even in those cases it is a challenging problem to construct operator algebras that contain the parametrices of elliptic elements. For higher singularities (e.g., when the base of the local cones has singularities of conical or edge type) some new problems appear for the first time, and it seems to be indispensable to launch a systematic approach in terms of "singular" operator algebras. During a conference in fall 2008 in Berkeley we gave an overview on the state of insight from the author's point of view, characterised by the key words "operator algebras on stratified spaces" (see the preprint "The iterative structure of corner operators". arXiv:0905.0977v1[math.AP]).

In the present talk we come back to one of the important details, namely, new norm growth estimates of families of edge operators, where the parameter plays the role of covariables in a new higher pseudo-differential calculus on an infinite

cone where the base may have singularities. The difficulty is to understand the behavior of such operators at the conical exits to infinity, taking into account a corner-degenerate behavior of the families in the parameter. We establish algebras of operators with such amplitude functions where the calculus is possible thanks to certain norm growth estimates which substitute what is known in scalar pseudo-differential theories as symbolic estimates.

Andras Vasy

Title: **Wave propagation on asymptotically De Sitter and Anti-de Sitter spaces.**

Abstract: In this talk I describe the asymptotics of solutions of the wave equation on asymptotically De Sitter and Anti-de Sitter spaces. This is part of a larger program to analyze hyperbolic equations on non-product, non-compact manifolds, similarly to how various types of 'ends' have been studied for the Laplacian and other elliptic operators on Riemannian manifolds. Part of the talk is on current work in progress.

Gregory Verchota

Title: **Linear elliptic operators requiring indefinite terms in the quadratic Dirichlet form in order for a full coercive estimate to hold.**

Abstract: Certain 4th order linear real constant coefficient elliptic differential operators $L = \sum_{|\alpha|=|\beta|=2} a_{\alpha\beta}\partial^{\alpha+\beta}$ are shown to satisfy a coercive integro–differential estimate

$$c\sum_{|\alpha|\leq 2}\int_\Omega |\partial^\alpha u|^2 dX \leq \sum_{|\alpha|=|\beta|=2}\int_\Omega a_{\alpha\beta}\partial^\alpha u\,\overline{\partial^\beta u} dX + c_0\int_\Omega |u|^2 dX,\ \ (c>0)$$

over the full Sobolev space $W^{2,2}(\Omega)$ only when the right side contains quadratic terms that are indefinite, in fact negative definite on an infinite dimensional subspace of $W^{2,2}(\Omega)$. These terms are shown to be necessary even when L, in addition, can be written as a sum of squares of homogeneous 2nd order operators $\sum p_j^2(\partial)$, so that L also has formally positive forms $\sum_j \int_\Omega |p_j(\partial)u|^2 dX$. In these cases all formally positive forms are shown to be noncoercive over $W^{2,2}(\Omega)$.

Boris Vertman

Title: **The regular-singular Sturm-Liouville operators and their zeta-determinants.**

Abstract: Recent advances in the computation of zeta-determinants for Laplace-type operators with specific regular-singular potentials of model type and general boundary conditions at the singularity have been made by Klaus Kirsten, Paul Loya and Jinsung Park. A formula for zeta-determinants for a general class of regular-singular potentials, however only for specific boundary conditions at the singular end, is due to Matthias Lesch.

This poses the question whether appropriate results can also be achieved for Sturm-Liouville operators with general regular-singular potentials and general boundary conditions. We answer this question affirmatively and provide a formula for the

zeta-determinant in terms of the Wronski-determinant of the boundary value problem, generalizing the earlier results of Lesch and Kirsten-Loya-Park.

This is a joint project with Matthias Lesch.

Ingo Witt

Title: **Symbolic parametrix construction for the strictly hyperbolic Cauchy problem.**

Abstract: The solution u to the strictly hyperbolic homogeneous Cauchy problem $Pu = f \equiv 0$ on X and $\gamma_k u = g_k$ on X_0 for $0 \leq k < \mu$ can be written as $u = \sum_{k=1}^{\mu} E_k g_k$, where each E_k is a sum of Fourier integral operators. Furthermore, it is known that real-principal type operators admit parametrices the kernels of which are one-sided paired Lagrangian distributions. In this talk, by identifying the Lagrangian submanifolds of $T^*(X \times X) \setminus 0$, $T^*(X \times X_0) \setminus 0$, and so on that arise, e.g., in compositions and by identifying the principal symbols on these Lagrangian submanifolds of the operators involved, we put both constructions together and come up with a calculus in which the strictly hyperbolic, but now inhomogeneous Cauchy problem (i.e., the above problem, where $f \not\equiv 0$ is possible) appears as an operator with an invertible principal symbol which allows a parametrix within the calculus.

M. W. Wong

Title: **Spectral invariance and essential equivalence of pseudo-differential operators with exit at infinity.**

Abstract: Based on the works of B.-W. Schulze, we prove the spectral invariance of pseudo-differential operators with exit at infinity on $L^p(\mathbb{R}^n)$, $1 < p < \infty$. Using a version of the Brown-Douglas-Fillmore theorem, we give a class of essentially equivalent pseudo-differential operators with exit at infinity on $L^2(\mathbb{R}^n)$.