SINGULAR ANALYSIS AND SPECTRAL THEORY OF PARTIAL DIFFERENTIAL EQUATIONS

AMS Meeting in Newark (DE), April 2-3, 2005

Constantin Bacuta

Title: **Regularity estimates for the biharmonic problem.**

Abstract: We consider the biharmonic Dirichlet problem on a polygonal domain. Regularity estimates in terms of Sobolev norms of fractional order are proved. The analysis is based on subspace interpolation theory. We apply our results to establish regularity estimates for the Stokes system and the Dirichlet plane elasticity system on polygonal domains.

Klaus Kirsten

Title: **Functional determinants of Laplace like operators.**

Abstract: We present techniques for the analysis of functional determinants for a certain class of Laplace like operators. Examples considered in detail are the Laplacian on the generalized cone and when spherically symmetric potentials are present. On the bounded generalized cone the functional determinant is expressed through quantities on its base. For balls and monopoles of any dimension these results can be used to find closed answers in terms of the Barnes or the Riemann zeta function. In the presence of a spherically symmetric potential the determinant is expressed through the Jost function for the corresponding scattering problem. Answers that allow for a numerically convenient evaluation are found.

Thomas Krainer

Title: **Resolvents of elliptic cone operators.**

Abstract: We study properties of the spectrum and the resolvent of closed extensions of general elliptic cone operators. In particular, we prove the existence of sectors of minimal growth under natural ellipticity conditions on the principal symbols of the operator. Special attention is devoted to the clarification of the analytic structure of the resolvent. The results presented are accessible at arXiv.org under math.AP/0410178 and math.AP/0410176.

Matthias Lesch

Title: **The Calderón projector for the Hessian of the perturbed Chern-Simons function on a 3-manifold with boundary.**

Abstract: The Morse theory of the Chern–Simons function leads to the construction of topological invariants, notably Taubes' construction of the Casson invariant and Floer's instanton homology. To obtain a Morse function the Chern–Simons functional has to be suitably perturbed. The Hessian then is a perturbation of the the odd signature operator by a nonlocal bounded operator.

In view of Heegard decompositions one also has to deal with this perturbed operator on manifolds with boundary. In my talk I will discuss the Calderon projector of the perturbed odd signature operator on a 3-manifold and point to possible applications. The talk is based on joint work with B. Himpel and P. Kirk.

Paul Loya

Title: **Eta invariants for first order regular singular operators.**

Abstract: The eta invariant originally appeared as the boundary correction term of the index formula of Atiyah, Patodi, and Singer, but since this seminal paper, the study of the eta invariant has taken off with a life of its own. In this talk, I will discuss joint work with Jinsung Park on eta invariants of first order regular singular operators on manifolds with boundary which generalize the Gauss-Bonnet and signature operators on conic manifolds. Such operators admit many self-adjoint extensions parametrized by Lagrangian subspaces of an associated finite-dimensional symplectic vector space. I will talk about the dependence of the eta invariant upon the choice of Lagrangian subspace and relate the eta invariant to a corresponding eta invariant of a smooth (nondegenerate) operator with global pseudodifferential boundary conditions on a compact manifold with boundary obtained by removing a neighborhood of the singularity.

Patrick McDonald

Title: **Zeta regularized determinants for conic manifolds.**

Abstract: We study relative zeta regularized determinants of Laplace type operators on compact conic manifolds. We establish gluing formulae for such determinants. For selfadjoint extensions of the Laplace-Beltrami operator corresponding to Dirichlet and Neumann conditions, we express the corresponding relative zeta regularized determinants as a ratio of determinants of certain finite matrices.

Irina Mitrea

Title: **Mixed boundary value problems on polygons.**

Abstract: We discuss well-posedness results for mixed Dirichlet-Neumann boundary value problems on polygonal domains in the plane with data in Sobolev spaces.

Marius Mitrea

Title: **Recent progress in the study of Navier-Stokes equations in rough domains.**

Abstract: I will discuss some recent work, in collaboration with S. Monniaux, on the structure of the domain of fractional powers of the Stokes operator in domains with rough boundaries. These results are then used to adapt the Fujita-Kato theory to the case of arbitrary Lipschitz domains in the three-dimensional Euclidean space.

Victor Nistor

Title: **Well posedness and regularity for the Laplace equation on polyhedral domains.**

Abstract: We prove that the inhomogeneous Laplace equation $\Delta u = f$ with Dirichlet boundary conditions has a unique solution in the *weighted* Sobolev spaces $K^{m+2}_{a+2}(P)$, provided that the data f is in $K^m_a(P)$ and $|a+1| < \eta$, where $m \in Z_+$ and $\eta > 0$ is a constant depending only on the polyhedral domain P. The weight is the distance to the faces of dimension $n-2$, that is, the distance to the singular points of the boundary. This result is due to Kozlov, Mazya, and Rossman in dimension $n = 2$.

Antonio Sa Barreto

Title: **Support theorems for radiation fields on asymptotically Euclidean manifolds.**

Abstract: We will discuss support theorems for a generalization of the Radon transform to asymptotically Euclidean manifolds.

Grigore Raul Tataru

Title: **Szegő projections on cosphere bundles of asymptotically Euclidean manifolds.**

Abstract: The cosphere bundle $\mathbb{S}^*X$ of an asymptotically Euclidean manifold X carries a contact structure and a CR structure with special degeneracies at the boundary. A class of parabolic (Heisenberg) pseudodifferential operators on $\mathbb{S}^*X$ is introduced and contains the Szegő projection associated to the CR structure. Questions regarding the relation between the scattering wavefront set of distributions on X and the singular support of corresponding CR distributions on $\mathbb{S}^*X$ are also addressed.

Greg Verchota

Title: **Uniqueness and counter examples to uniqueness for the L_p oblique derivative problem.**

Abstract: The oblique derivative problem for harmonic functions is formulated with respect to a continuous transverse unit vector field defined on the boundary of a bounded Lipschitz domain that has connected boundary. Data is prescribed in Lebesgue spaces on the boundary with respect to surface measure. Data is taken on

almost everywhere with respect to surface measure by way of nontangential convergence. Solutions are taken from the class of harmonic functions with nontangential maximal function of the gradient in the Lebesgue space. This is the setup of a 1985 theorem of A. P. Calderón's that uses certain nonclassical layer potentials to solve the problem in L_2 modulo a finite number of linear conditions. In the plane for L_2 data solutions are shown to be unique up to constant solutions so that the number of linear conditions is in fact one. The same can be shown in higher dimensions as long as the vector field is Holder continuous. For $p < 2$ explicit curvilinear polygons and vector fields are constructed that possess any given number of explicit solutions with zero a.e. data. A Lipschitz domain is constructed with an infinite number of such solutions. Consequences for the potentials follow.

Ingo Witt

Title: **Solvability for a class of semilinear elliptic Fuchsian PDEs.**

Abstract: We discuss solvability for semilinear elliptic equations of the form

$$Au = F(x, B_1u, \dots, B_Ku) \text{ in } X \setminus \partial X, \quad Tu = g \text{ on } \partial X,$$

for a differential operator A that is Fuchsian with respect to ∂X, for a C^∞ compact manifold X, and that together with the boundary condition $Tu = 0$ is supposed to be positive definite selfadjoint in the weighted L^2 space $H^{0,\delta}(X)$, for some $\delta \in \mathbb{R}$. The Fuchsian differential operators $B_1, \dots, B_K$ are of orders strictly less the order of A, and the nonlinearity $F = F(x, \nu)$ is of at most polynomial growth in ν. Moreover, the linear surjective boundary map $T\colon D_+ \to \mathbb{R}^\mu$ factors through D_+/D_-, where D_+ and D_- are the maximal and minimal domains of A in $H^{0,\delta}(X)$, respectively; $\dim D_+/D_- = 2\mu$. As solutions to the above problem are unbounded in general, the main step consists in an *a priori* description of the asymptotics of these solutions $u = u(x)$ as $x \to \partial X$.