SPECTRAL GEOMETRY OF MANIFOLDS WITH BOUNDARY AND SINGULAR SPACES

AMS Meeting in Miami, April 1-2, 2006

David Borthwick

Title: **Asymptotically hyperbolic manifolds with computable resonances.**

Abstract: Examples where resonances are computable are notoriously difficult to find. We study a family of asymptotically hyperbolic metrics given as warped products, for which the resonance set is computable in terms of the discrete spectrum of a compact base manifold. *Received:* February 07, 2006

Juan Gil

Title: **On rays of minimal growth for elliptic cone operators.**

Abstract: We present an overview of some recent results on the existence of rays of minimal growth for elliptic cone operators and two new results concerning the necessity of certain conditions for the existence of such rays. *Received:* February 07, 2006

Daniel Grieser

Title: **Eigenvalue estimates, isoperimetric inequalities and flows in networks.**

Abstract: We give a new way of looking at Cheeger's inequality which gives a lower bound for the first eigenvalue of the Laplacian. This involves a continuous analogue of the classical max flow min cut theorem in discrete network theory. This gives a new proof of Cheeger's inequality and of the Hayman/Osserman inequality which bounds the first Dirichlet eigenvalue of a plane simply connected domain from below in terms of the inradius. *Received:* February 07, 2006

Klaus Kirsten

Title: **ζ-regularized determinants on manifolds with conical singularities.**

Abstract: As has been noticed recently (work by K. Kirsten, P. Loya and J. Park), ζ-functions associated with general self-adjoint extensions of Laplace-type operators over conical manifolds have, in general, countably many logarithmic branch cuts on the non-positive real axis and unusual locations of poles with arbitrarily large multiplicity. For example, at $s = 0$ the ζ-function might have a pole of order one as well as logarithmic singularities. As a consequence, the standard ζ-regularized determinant will in general not be well-defined. However, a natural prescription is to subtract the singularities and consider the determinant related to the resulting 'regularized' ζ-function. This is the procedure employed and closed answers are found showing explicitly the dependence of the determinants on the self-adjoint extensions considered. For cases when the ζ-function is analytic at $s = 0$, standard results are recovered. *Received:* January 26, 2006

Thomas Krainer

Title: **Elliptic boundary value problems on manifolds with polycylindrical ends.**

Abstract: We investigate general elliptic operators subject to Shapiro-Lopatinsky elliptic boundary conditions on manifolds with polycylindrical ends. We derive Fredholm criteria and regularity results in appropriate Sobolev spaces. This is achieved by compactifying the manifold with polycylindrical ends to a manifold with corners. The operators then turn out to be so called "cusp operators", i.e. they are near corner points of codimension q generated by the C^∞-functions and the vector fields of the form $x_l^2\partial_{x_l}$, $l = 1, \ldots, q$, and ∂_{y_j}, $j = 1, \ldots, n-q$, where $y = (y_1, \ldots, y_{n-q})$ is the variable in an open set of $\mathbb{R}^{n-q}$, and $x = (x_1, \ldots, x_q) \in \overline{\mathbb{R}}_+^q$. Without the presence of boundary conditions, such operators were considered by Richard Melrose and Victor Nistor in 1996 in the context of index-theoretical investigations on manifolds with corners of codimension 1 (unpublished), and in the case of higher codimensions by Robert Lauter and Sergiu Moroianu (2002). The preprint of the material presented in this talk is accessible from http://arXiv.org under math.AP/0508516 (2005). *Received:* February 04, 2006

Paul Loya

Title: **The Calderon projector for manifolds with corners of codimension two.**

Abstract: The Calderon projector for Dirac operators on manifolds with smooth boundary completely determines the Fredholm properties of boundary value problems for the Dirac operator. For manifolds with corners, the Calderon projector exists (even for manifolds with Lipschitz domains) as a singular integral operator, but not a lot has been studied in regards to its fine structure. For manifolds with corners of codimension two, it turns out that if all the corners are "blown-up", then the Calderon projector has a well-defined meaning on a space of functions with asymptotics. Moreover, it describes the Fredholm properties for boundary value problems of the Dirac operator on a corresponding space of functions with asymptotics and it can be "explicitly" described as an element of a type of Boutet de Monvel class of operators mapping between manifolds with boundary. In this talk I will discuss these results and results for manifolds with corners of higher codimension. *Received:* February 07, 2006

Patrick McDonald

Title: **The eta invariant for quantum graphs.**

Abstract: Let Γ be a compact quantum graph with natural coordinate x associated to each edge. Let D be the Dirac operator defined initially on functions smooth along the edges of Γ via differentiation with respect to the natural coordinate. We parameterize self-adjoint extensions of D using a collection of unitary matrices determined by the structure of Γ. We compute the eta invariant associated to a self-adjoint extension of D in terms of the unitary matrix representing the given extension. *Received:* February 07, 2006

Gerardo Mendoza

Title: **On eigenspaces, hypoellipticity, and positivity of $\mathbb{R}$-invariant differential operators..**

Abstract: Let M be a compact manifold, let T be a globally defined real smooth nowhere vanishing vector field on M, and let $\mathfrak{a}_t$ denote the one-parameter group of diffeomorphisms generated by T. Suppose there is a T- invariant positive density on M. Let $E \to M$ be a Hermitian vector bundle and suppose there is a one-parameter group of isometries $\mathfrak{a}_t^* : E \to E$ covering $\mathfrak{a}_{-t}$. Let $L_T \in \mathrm{Diff}^1(M, E)$ be the differential operator associated with this action. Suppose $P \in \mathrm{Diff}^m(M; E)$ commutes with L_T. We will

outline proofs of the following two statements. (1) If $\sigma(P) + \sigma(-iL_T)^m - \lambda I$ is invertible for λ in a real line in $\mathbb{C}$ on $T^*M \setminus 0$ then $-iL_T : C^\infty(M;E) \cap \ker P \to C^\infty(M;E) \cap \ker P$ has a selfadjoint Fredholm extension $L_T : \mathcal{D} \subset L^2(M;E) \cap \ker P \to L^2(M;E) \cap \ker P$, so it has discrete spectrum contained in $\mathbb{R}$. (2) If in addition P is hypoelliptic on $\{\nu \in \mathrm{Char}(P) : \sigma(-iL_T)(\nu) > 0\}$, then the spectrum of $-iL_T$ has only finitely many negative elements. Time permitting, we will give applications to the analysis of the b-spectrum of an elliptic b-complex. *Received:* February 06, 2006

IGOR PROKHORENKOV

TITLE: **Witten approach to Morse inequalities on manifolds with boundary and generalizations.**

ABSTRACT: We will start by explaining recent work of B. Helffer and F. Nier on Witten deformation for manifolds with boundary. We will show how their methods could be generalized to obtain semiclassical asymptotics of the eigenvalues of the Witten Laplacian. The main novelty is that the model operator on a manifold with boundary can have both discrete and continuous spectrum. *Received:* February 06, 2006

JÖRG SEILER

TITLE: **H_∞-calculus for differential operators on conic manifolds with boundary.**

ABSTRACT: I want to describe conditions that ensure that realizations (subject to homogeneous differential boundary conditions) of elliptic differential operators on manifolds with conical singularity possess a bounded H_∞-calculus. *Received:* February 03, 2006

INGO WITT

TITLE: **Branching of conormal asymptotics along edges.**

ABSTRACT: To guarantee well-posedness for PDEs near edge singularities, conditions on the solutions are to be imposed along the edges, too. To do so, one needs an *a priori* good understanding of the possible conormal asymptotic expansions of the solutions to the PDEs under investigation near those edges. Here we classify the possible asymptotic expansions into asymptotic types and discuss both algebraic and order-theoretic characterisations of the latter. Compared to the situation when only conical singularities are present, the appearance of branch points for the asymptotics as the edge parameters vary constitutes an additional difficulty. The given classification is minimal in the sense that the resulting lattice of asymptotic types is generated by the proper asymptotic types, i.e., those asymptotic types that are annihilated by elliptic holomorphic Mellin symbols. This minimality is crucial for applications to non-elliptic as well as non-linear elliptic problems. More specifically, for a given proper asymptotic type we construct an elliptic holomorphic Mellin symbol realising this asymptotic type in the sense of annihilating it thus proving equivalence between the provided intrinsic characterisation of asymptotic types and their "intuitive meaning". *Received:* February 07, 2006